S-Q 1997 - Final Exam
Edited for 1998

Some of the questions have been removed, because they are for topics we did not cover this year, or are very similar to questions that have already appeared on midterms or problem sets this year.

The actual test contained several additional problems. This test, as it is currently written, should take 2 hours or less to complete.

1. (21 points) Indicate whether each of the statements below is true or false. For full credit, you must provide a brief explanation of each answer.

True False  The Stack ADT can be implemented by an ordinary C array in such a way that the Push, Pop, isEmpty, and Initialize operations all execute in time $O(1)$.

True False  The load factor of a hash table that uses separate chaining can be greater than 1.0.

True False  Finding the largest element in an unsorted array of $N$ integers requires time $\Omega(N)$.

True False  Mergesort requires time $O(N \log N)$ on arrays that are already sorted, where $N$ is the length of the array.

True False  Bubblesort runs in $O(N)$ time if the input array contains fewer than $N$ inversions.

True False  Searching for a key in a trie requires at most $N$ comparisons, where $N$ is the number of keys in the trie.

True False  In the level-order tree traversal algorithm that uses a queue, the length of the queue cannot possibly exceed $\frac{N}{2} + 3$, where $N$ is the number of nodes in the tree.

2. (14 points) Give the algorithms for performing the following Queue ADT operations, if the queue is implemented as a circular, singly-linked list, as illustrated by the data structure shown below:
typedef struct _queue_t {
    struct _queue_t *next;
    int value;
} queue_t;

(a) DEQUEUE
(b) ENQUEUE

3. (8 points) Let $G = \langle V; E \rangle$ be a directed graph, where

$$V = \{A, B, C, D, E\}$$
$$E = \{<A, B >, <E, B >, <B, C >, <A, D >, <C, D >, <A, E >\}$$

(Recall that the edge $<A, B>$ is a directed edge from vertex A to vertex B.)

(a) Draw the graph $G$.
(b) Is $G$ connected, and if so in what manner?
(c) Does $G$ have any cycles? If so, what vertices are joined by a cycle?

4. (12 points) Show the steps that would be taken by a topological sort of the directed graph shown below. (Note that there may be more than one correct topological order for this directed graph, but you need only find one.)

5. (15 points) Show the steps that would be taken by Kruskal’s algorithm to find the minimum spanning tree for the network shown below. Be sure to show the final tree.
6. (15 points) Give an algorithm to determine whether a binary tree (implemented by the data structure shown below) has the min-heap order property. The result of the algorithm must be YES if the tree has the min-heap order property, and NO otherwise.

```c
typedef struct _tree_t {
    struct _tree_t *left;
    struct _tree_t *right;
    int key;
} tree_t;
```

7. (15 points) Show that any directed graph which does not contain any cycles must contain at least one vertex that has no out-arc.

8. (5 points) Briefly describe the Rabin-Karp string searching algorithm. (Use pseudo-code if appropriate, but you do not need to write a complete C implementation.)

9. (10 points) Carol, Doug, and Edwin are having a heated discussion about string searching. Carol says that Boyer-Moore is always the fastest string searching algorithm, while Doug claims that Rabin-Karp is always faster. Edwin says that it depends on the length of the pattern and the length of the text being searched. Who is correct? Support your claim.